Large-Signal Feedback Control of a Bidirectional Coupled-Inductor Čuk Converter

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Abstract—Under conditions of order reduction, a nonlinear control of a bidirectional coupled-inductor Čuk converter suitable for large-signal applications is presented. The converter is accurately modeled as a second-order bilinear system and the conditions established for local controllability. The integration of converter state equations and the subsequent introduction of a linear recurrence between the output variable and an external reference signal lead to a nonlinear control law that is implemented by means of an analog divider, standard operational amplifiers, and a pulse-width modulator. As a result, the output variable follows proportionally the reference signal, this allowing the obtention of different types of power waveforms in the converter output. Experimental results verify the theoretical predictions.

I. INTRODUCTION

SWITCHING converters can be represented taking part of a class of feedback systems as illustrated in Fig. 1. The basic operation of the regulator loop consists in comparing the output variable with a reference input to generate an error signal, which, after being appropriately filtered and amplified, results in a continuous control signal. This is eventually transformed by means of a modulator into a discrete variable defining the switch duty cycle. The main difficulties in the analysis of switching regulators are due to the highly nonlinear nature of dc-to-dc converter systems.

To solve this problem, different linearizing modeling techniques were proposed in the past, their validity limits being located in a small neighborhood of the steady-state operating point. However, the analysis becomes more difficult beyond these limits, when large-signal behavior is required. This case is particularly important either when designing a switching converter-based servosystem using a variable reference or when, in a conventional regulator, a load perturbation appears. The dynamical analysis becomes also rather complex in the discontinuous conduction mode where the instant in which the switch current becomes zero depends on the converter state and not on its duty cycle. Therefore, two constraints expressing the current cancellation must be added to the three piecewise-linear vector differential equations. First, the switch current takes the same value at both the beginning of the first subinterval and the end of the second one; second, the switch current is zero during the third subinterval of the switching period. Moreover, the existence of an additional nonlinearity nonrelated to the control variables but to the converter state variables makes the design of a feedback loop suitable for large-signal operation particularly difficult. In order to avoid the discontinuous conduction mode and, at the same time, allow the existence of light load levels, the bidirectional switch was introduced, this having led to a bilinear description of the switching converter [1], [2].

The transformation of the canonical buck, boost, or buck-boost structures into bidirectional cells has extended the dynamic performances of the power converter as the time constant of the output filter is strongly affected by the low-impedance input network. On one hand, the well-known linear control strategies cannot be used in such bidirectional converters since large-signal transients are often present in the current flow reversal. Therefore, in such converters, large-signal control becomes mandatory, this having to take into account the nonlinearities introduced by the sampling process as well as the additional constraints that are needed to obtain the desired closed-loop regulation. On the other hand, a large-signal control technique in bidirectional cells has provided excellent performances in the elementary converters as reported in [1], [2]. This approach is now applied to control a complex converter such as the bidirectional coupled-inductor Čuk converter, shown in Fig. 2, under conditions of order reduction.

In such a converter, a reduction of a fourth two second-order can be achieved if two conditions are fulfilled. First, if the magnetic coupling satisfies the matching...
condition \( n = k \), zero current ripple will be obtained at the output and the need for output capacitor \( C_2 \) will be completely eliminated as it was reported by Čuk in [4], [5]. The elimination of this capacitor results in a simplified and extremely favorable loop-gain dynamics when this converter is used in push–pull configuration, thus implementing a push–pull switching audio power amplifier [4]–[6]. Second, an additional order reduction can be obtained if the magnetic coupling is perfect, i.e., \( k = 1 \), which results in only one equivalent inductor. Thus, this paper covers the analysis and design of a nonlinearly controlled bidirectional second-order converter suitable for large-signal applications.

II. BILINEAR REPRESENTATION AND LOCAL CONTROLLABILITY

Fig. 3 shows an equivalent circuit of a bidirectional coupled inductor Čuk converter with the condition \( n = k = 1 \). Transistors and diodes have been assumed to be ideal, therefore no parasitics or storage-time modulation effects have been considered.

Because of the bidirectional current capability of the switch, the discontinuous conduction mode does not take place and therefore the switching converter can be represented by two piecewise-linear vector differential equations:

\[
\dot{x} = A_1 x + b_1 V_s \quad \text{for} \quad t \leq T_{on} \quad (1)
\]

\[
\dot{x} = A_2 x + b_2 V_s \quad \text{for} \quad T_{on} \leq t \leq T \quad (2)
\]

where \( X \) is the power-stage state vector and \( V_s \) is the dc input.

Equations (1) and (2) can be combined in only one bilinear expression:

\[
\dot{x} = (A x + a) + (B x + b) u \quad (3)
\]

where

\[
u = \begin{cases} 1 & \text{for} \quad t \leq T_{on} \\ 0 & \text{for} \quad T_{on} \leq t \leq T \end{cases}
\]

\[
x = \begin{bmatrix} i_m \\ v_1 \\ \end{bmatrix}
\]

and matrices \( A, a, B, \) and \( b \) are given, respectively, by

\[
A = A_2 = \begin{bmatrix} 0 & -1/L_M \\ 1/C_1 & -1/C_1 R \end{bmatrix}
\]

\[
a = b_2 V_s = \begin{bmatrix} 1/L_M \\ 1/RC_1 \end{bmatrix} V_s
\]

\[
B = A_1 - A_2 = \begin{bmatrix} 0 & 1/L_M \\ -1/C_1 & 0 \end{bmatrix}
\]

\[
b = [b_1 - b_2] V_s = 0
\]

where \( 0 \) is the null matrix.

The bilinear system described by (3) is locally controllable since the dimension of the Lie algebra generated by vectors \( Ax + a \) and \( Bx + b \) is 2. To prove that [see Appendix], it can be shown after tedious calculations that the Lie
algebra contains the vectors

\[ W_1 = Bx \]
\[ W_2 = [Ax + a, Bx] \]
\[ W_3 = [Ax + a, (Ax + a, Bx)] \]

which are related among them as follows.

I) \( W_1 \) and \( W_2 \) are linearly independent in the complement of conic.

\[ C_{12} = \{(i_{m,1}, v_{i_1}) - 2L_Mi_{m,1} + L_MV_e^{i_{m,1}} - RC_iV_e^{i_{m,1}} = 0\} \]

II) \( W_1 \) and \( W_3 \) are linearly independent in the complement of conic.

\[ C_{13} = \{(i_{m,1}, v_{i_1}) - 2(2RL_M) - v_1^{i_{m,1}}(2RC_i) + i_{m}^{(R^2C_iV_e^{i_{m,1}} + L_MV_e^{i_{m,1}} + v_1^{3RC_iV_e^{i_{m,1}}}) = 0\} \]

III) \( C_{12} \cap C_{13} \) represents a point in \( \mathbb{R}^2 \) where \( W_2 \) and \( W_3 \) are linearly independent.

III. NONLINEAR CONTROL OF THE COUPLED INDUCTOR CUK CONVERTER WITH THE CONDITION \( n = k = 1 \)

Equations (1) and (2) can be solved using the following expression:

\[ x(t) = e^{A(T-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(T-t)}b_1 V_e d\xi \]

\[ i = 1 \text{ for } t \leq T_{on} \]
\[ i = 2 \text{ for } T_{on} \leq t \leq T \]
\[ t_{o1} = 0 \]
\[ t_{o2} = T_{on} \]

(10)

Considering the subinterval \( t_i < t < t_i + \tau \)

(10) becomes

\[ x(t_i + \tau) = x(t_i) + \int_{t_i}^{t_i + \tau} e^{A(T-t)}b_1 V_e d\xi \]

\[ = [\Box + A_1\tau x(t_i) + b_1 V_e A_1^{-1}e^{A_1\tau} - \Box] \]

(11)

where \( \Box \) is the unit matrix and the exponential matrix has been approximated by a first-order Taylor series expansion.

Similarly, for \( t_i < t < t_i + \tau \) it can be derived that

\[ x(t_i^{n+1}) = e^{A(T-t_0)}x(t_i^{n+1}) + \int_{t_i^{n+1}}^{t_i^{n+1} + \tau} e^{A(T-t)}b_1 V_e d\xi \]

\[ = e^{A_1\tau [A_2^{-1}\Box - e^{A_1\tau}b_1 V_e] + x(t_i^{n+1}) + (A_1 - A_2)x(t_i^{n+1})} \tau \]

(12)
where

\[
H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{T}{L} \\ \frac{T}{C} & 1 - \frac{T}{RC} \end{bmatrix}
\]

\[
F = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} \frac{T}{LC} & \frac{1}{T} \\ -\frac{1}{C} + \frac{RC}{T^2} & \frac{T}{LC} \end{bmatrix}
\]

\[
G = \begin{bmatrix} g_{11} \\ g_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{L} - \frac{T^2}{RCL} & V_i \\ \frac{T^2}{LC} + \frac{T}{RC} - \frac{T^2}{R^2C^2} & V_i \end{bmatrix}
\]

Finally, introducing in (15) the following control recurrence:

\[
v_i(t_{i+1}) - C_i = W[v_i(t_i) - C_i]
\]

where \(C_i\) represents an external reference and \(W\) is a positive arbitrary constant less than 1, results in

\[
\tau(t_i) = -g_{12}h_{21}i_m(t_i) + v_i(t_i)(W - h_{22}) + C_i(1 - W)
\]

\[
f_{11}i_m(t_i) + f_{22}v_i(t_i)
\]

(18)

This nonlinear control law is represented by the block diagram depicted in Fig. 4 and has been implemented as shown in Fig. 5, where the complete bidirectional converter and its control circuit are depicted in detail.
Fig. 6(a) and (b) shows the regulation up and down of the prototype, respectively. Fig. 7, in turn, represents the obtained efficiency versus the output current. On the other hand, Fig. 8 illustrates the excellent dynamic behavior of the regulation by following different types of variable reference. The measured bandwidth is 714 Hz.

Finally, Fig. 9 shows the output voltage behavior when using a sinusoidal reference and the load has pulsating characteristics, this being implemented as represented in Fig. 10. The output voltage exhibits in this case a fast recovery, i.e., only 5% of a half a cycle time and negligible overshoot 10%.

IV. CONCLUSIONS

The obtained results show the feasibility of a large-signal control of PWM complex converters. The technique presented in the paper can be used to implement high-performance sinusoidal inverters or power amplifiers. It
must be remarked that the developed prototype has been designed for low-power applications since our main interest was to verify the validity of the proposed control loop. The control procedure can also be easily applied to high-order converters such as the fourth-order bidirectional Cuk converter as reported in reference [13]. In all cases, the system performances are basically limited by the characteristics of the analog multiplier. In that sense, further research is needed in order to simplify the number of elements in the feedback path by implementing some of the analog functions by means of digital circuits.

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APPENDIX

Considering the vector fields \( V \) and \( W \) whose coordinates are \( V(x) = Ax + a \) and \( W(x) = Bx \), respectively, in the usual base of \( \mathbb{R}^n \), the Lie derivate of the field \( W(x) \) along \( V(x) \) is defined as

\[
L^W_V(x) = [V, W] = \frac{\delta W}{\delta x} V - \frac{\delta V}{\delta x} W
\]  

or, in coordinates,

\[
[V, W] = B(Ax + a) - A(Bx).
\]  

The bilinear system described by (3) will be controllable if and only if the Lie algebra generated by operators \( V \) and \( W \) is of full rank in any operating point of the converter [12]. Since, in this case, we analyze a second-order system, the Lie algebra will be of full rank if its dimension is 2.

In order to investigate a base of this algebra, we define the set of vectors

\[
\{W_1, W_2, W_3, \ldots \}
\]

where

\[
W_1 = W = Bx
\]

\[
W_2 = [V, W] = [Ax + a, Bx]
\]

\[
W_3 = [V, [V, W]] = [Ax + a, [Ax + a, Bx]]
\]

\[
\vdots
\]
If two vectors of this set are linearly independent, the model will be controllable. On the other hand, the model will be uncontrollable if the following determinants are zero:

$$\det(W_1, W_2) = 0$$  \hspace{1cm} (A7)
$$\det(W_1, W_3) = 0$$  \hspace{1cm} (A8)
$$\det(W_2, W_3) = 0$$  \hspace{1cm} (A9)

The analysis of this set of equations reveals that we can always find two linearly independent vectors since $W_1, W_2, \ldots, W_5$ are related among them as follows:

I) $W_1$ and $W_2$ are linearly independent in the complementary of conic:

$$C_{1}\equiv\{(i_m, v_1) \mid -2Lmi_mv_1 + L V_i - RCv_1v_1 = 0\}$$

II) $W_1$ and $W_3$ are linearly independent in the complementary of conic:

$$C_{2}\equiv\{(i_m, v_1) \mid 2Lmi_mv_1 + 2RCv_1v_1 = 0\}$$

III) $C_{12} \cap C_{13}$ represents a point in $\mathbb{R}^2$ where $W_2$ and $W_3$ are linearly independent.

REFERENCES


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